MEMORANDUM

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FROM: R. H. Lyon

SUBJECT: The Role and Limitations of Damping in the Control of Vibration and Noise

REFERENCE: A talk presented to the Acoustical Society of America at its 65th meeting in New York City, May 15-May 18, 1963, by Richard H. Lyon, Bolt Beranek and Newman Inc., Cambridge, Massachusetts

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I. Introduction

In the scheme of things, it is apparently appropriate that a person who has never been actively involved in the study of damping should be called upon to discuss how and when others should apply damping to structures. My own concern with damping is with the loss factor as a parameter which occurs in my equations. It is going to be my role here to explain when and how I believe an increase in the loss factor can produce a significant reduction in the vibratory response and sound radiation of structures excited by acoustic noise, boundary layer turbulence, impact noise, rain on the roof, etc. The discussion will not, of course, be complete, but we hope to encompass a broad enough range of possibilities so that at least general trends are apparent in the answers we arrive at.

Here, rather than merely to ask whether damping will reduce vibration under some particular method of forcing, we shall include environmental effects of correlation of areas of sources and loading effects of the medium. In assessing

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whether useful amounts of damping may be expected to be achieved in any particular application, we idealize the source correlation effects by considering two extreme cases: excitation by acoustic noise where the correlation lengths are tied directly to the frequencies through the acoustic wavelength, and by point excitation which broadly includes impact, turbulent boundary layer, and rain on the roof noise. We shall consider the frequency and wavenumber response of panels, the properties of the source, correlation in space and time, and the radiation behavior of the structure to say whether damping may be expected to be useful in reducing the vibration response and/or the acoustic radiation from the structure.

We do this by studying how these forms of excitation will affect the so-called free and forced response of the structure and whether the resulting radiation from the structure is dominated by the free or forced motion. In addition, we will develop simple relations between the frequency, loss factor, and mean-free path on the structure to determine whether one should consider it as a bounded or an unbounded system, both from the standpoint of vibratory response and acoustic radiation behavior.

II. Response Regimes of the Simple Resonator

In Fig. 1, we have plotted the mean square displacement response of a simple resonator to a pure tone force as a function of frequency. At very low frequencies, the response is governed by the stiffness element and is independent of frequency. Also, since the stiffness is controlling the response in this regime there is little dependence of the response amplitudes on the size of the dissipation or damping element. At very high frequencies, the response is governed by the mass element in the resonator and the displacement decreases as the fourth power of the frequency of this regime. Again, in this region the response tends to be independent of the damping.

In the intermediate region, there is a range of frequencies for which the amplitude of response can become quite large, reaching a peak which is inversely proportional to the second power of the loss factor and having a bandwidth proportional to the loss factor. This regime we call the resonant part of the response. Pure tone or extremely narrow band excitation in this regime produces a mean square response inversely proportional to the second power of the loss factor. On the other hand, broadband noise, because it is being passed through a filter of bandwidth proportional to η, has a mean square response inversely proportional to the first power of the loss factor.

For this simple system, therefore, we see that the response in the stiffness for mass controlled regions of behavior is damping independent; and in the resonant regime will depend upon the bandwidth of the excitation. For broadband excitation, the response will be inversely proportional to the first power of the loss factor. This conversion between response to pure tone and broadband noise is an important factor to recall when one is using pure tone response or transmission characteristics to assess the effectiveness of damping in reducing the response amplitude. We might also note that the reduction in response to broadband noise cannot be reduced further by increasing the loss factor beyond $\eta \sim 1$, since beyond that point the stiffness controlled response dominates the response.

III. Vibration and Radiation of Unbounded Thin Plates

We shall first consider the response and radiation of an unbounded thin plate which is excited by boundary layer noise or acoustic radiation. We shall consider the response of this plate at a fixed frequency ω. At this frequency, we shall assume that the plate supports free waves with the wave-number k_p, as shown in Eq. (1).

$$k_p = \sqrt{\omega/\kappa c_\ell} \qquad (1)$$

ω = radian frequency

κ = radius of gyration for cross-section

c_ℓ = longitudinal plate velocity

The loss factor η is defined through the complex Young's modulus, as shown in Eq. (2).

$$E = E_o(1 - i\eta) \qquad (2)$$

E_o = Young's Modulus

Although we have defined the loss factor through the Young's modulus, we shall not restrict ourselves conceptually to such a loss mechanism but shall include in our thinking any form of dissipation such as edge absorption, applied damping, radiation losses, etc.

There are, of course, no infinitely extended plates in nature and therefore we must inquire what conditions on the plate dynamics and geometry are necessary so that we may consider a plate to be effectively unbounded. If we introduce the mean free path for waves on the plate, d, as defined in Eq. (3)

$$d = \pi A_p/P \tag{3}$$

A_p = plate area

P = perimeter

then the condition that a free wave which is generated shall die away before it reaches the boundary is essentially that the product $k_p\eta d$ shall be greater than unity, as shown in Eq. (4).

$$k_p\eta d > 1 \quad \text{for } \infty \text{ behavior} \tag{4}$$

$$\text{or, } \gamma \gtrsim 1 \text{ for edge absorption}$$

Expressed in terms of a boundary absorption, this is equivalent to saying that the boundary absorption coefficient γ shall be of the order of unity for infinite plate behavior, which means the absence of reflection and lack of reverberant build-up in the plate.

In Fig. 2, we have plotted the mean square displacement response of an infinitely extended plate versus excitation wavenumber at a fixed frequency ω. As in the case of the resonator, we have indicated three distinct regimes of dynamical behavior of the plate or the mechanical system. At small wavenumbers or long wavelengths, the plate is bent at a scale longer than that of the natural wavelength at that frequency. Accordingly, the response tends to be mass controlled because it is the inertia of the plate that retards the exciting force. At very high wavenumbers, or very small wavelengths of excitation, the stiffness of the plate becomes dominant in opposing the applied forces and the displacement response drops off rapidly with increasing wavenumber as k^{-8}. Near k_p, the free wavenumber, there is a regime of large response with bandwidth again proportional to η and amplitude proportional to η^{-2}. It is the response in this regime that we refer to as free wave response.

We have, in addition, drawn wavenumber spectra of the two forms of excitation which we suggested earlier. These are the acoustic wave excitations which have wavenumbers up to, but not exceeding, the acoustic wavenumber k_a and a point source excitation which has wavenumbers of excitation over a very broad range, generally including the free wavenumber k_p.

We are considering in Fig. 2 the case where $k_a < k_p$, or where the sound speed is greater than the flexural wave speed. These speeds are equal at the so-called critical frequency and this frequency forms an important dividing line between various forms of radiation and response behavior. It is dependent upon the properties of the fluid and the plate. For example, an 1/8" aluminum plate in air has a critical frequency of 3 kc corresponding to a wavelength of 4", whereas the critical frequency for a 2" steel plate in water is 5 kc with the wavelength of 1 ft.

When $k_a < k_p$, as shown here, the frequency is said to be below critical. Under these conditions, we note that acoustic wavenumbers do not excite the free waves, but instead all the acoustic response will be in a mass controlled region. The point source, on the other hand, may well have wavenumbers which encompass the free wave region and generate a large response which is damping dependent at the free wavenumber k_p. In this regime, therefore, we would conclude that the acoustic vibratory response would be damping independent but that the response to point source would be damping dependent, unless $\eta \sim 1$.

The only vibrations of the panel which radiate sound are those which have a wavenumber less than or equal to k_a. Thus, all the radiating wavelengths, whether excited by the point source or the sound field, are in the mass controlled regime. The acoustic radiation of the unbounded panel, therefore, regardless of the source of excitation, will be mass controlled and therefore damping independent. We arrive at quite different conclusions for the effectiveness of damping on the panel depending on whether we are primarily interested in the vibratory response or in the acoustic radiation properties. Another point to note is that the addition of stiffness to the structure would not be beneficial because it would not change the response in the mass controlled region and might only send the large vibration levels of the free wave regime down into the regime of large acoustic excitation and radiation, as we see in Fig. 3.

In Fig. 3, we see the relative positions of response and excitation spectra when the frequency is above the critical frequency or when k_a has become larger than k_p. In this instance, both the acoustic source and the point source can generate a large amplitude response in the free wave regime of a plate. Accordingly, we may expect primarily free wave response in the plate. Again, since the acoustic radiation is dominated by wavenumbers $\leq k_a$, we expect a highly excited and damping limited free wave response to be responsible for the sound radiation whether the excitation is acoustic or boundary layer turbulence, impact noise, etc.

Above the critical frequency for the unbounded plate, therefore, one finds that the vibratory response and the resulting radiation are determined by the total damping of the system. As we see in Eq. (5),

$$\eta = \eta_{int} + \eta_{rad} \tag{5}$$

$$\eta_{int} > \eta_{rad} \quad \Longrightarrow \quad \eta_{int} \frac{\rho_p}{\rho_f} \frac{6h}{\lambda_a} > \sigma_{rad}$$

ρ_p = plate density $\qquad \lambda_a$ = acoustic wavelength

ρ_f = fluid density $\qquad \sigma_{rad}$ = acoustic radiation efficiency.

h = plate thickness

the total damping is a combination for that due to the internal or applied damping and the radiation losses of the structure. In order for the internal or applied damping to be effective in reducing response in this case, it must exceed the radiation damping in magnitude. This condition is also shown in Eq. (5) and is expressed in terms of the internal loss factor, the ratio of densities of the plate to fluid medium, and the ratio of 6 times plate thickness to the acoustic wavelength. The product of these is to be compared to the radiation efficiency. For large plates above the critical frequency, the radiation efficiency is essentially unity and the right-hand side of the inequality becomes 1.

Consider again our 1/8" aluminum plate in air at a frequency of 4 kc. The internal loss factor according to Eq. (5) must be greater than 10^{-3} in order to be effective in reducing the damping. This is not a difficult level of damping to achieve. On the other hand, for the 2" plate in water at a frequency of 6 kc, the loss factor must be greater than 10^{-1}. This is a bit more difficult to achieve, particularly for a steel plate of 2" thickness. On this basis, we would say that it should be fairly simple to reduce the vibration and the acoustic radiation of the aluminum panel in air whereas it might be rather difficult to achieve effective damping levels for the steel plate in water, either above or below the critical frequency, as long as the product $kd\eta$ is greater than unity.

We can illustrate some of our conclusions on the effect of damping on the acoustic response and radiation of metal panels in air by reference to some experiments which have been carried out on the transmission loss of these panels. In Fig. 4, we show the measured transmission loss of a 90 mil aluminum plate having dimensions 12" x 34".

Below 1 kc, the TL is essentially mass controlled, as we would expect from our previous discussion. Above 1 kc, the behavior breaks away from this law, as shown by the solid dots and line denoted "Without Added Damping." Damping material was added to the panel and it was effective in reducing changing vibratory response to impact over the entire range of frequencies, as shown in the upper part of the figure. However, the increase in TL with damping only occurs above the region of mass law response, as shown by the open circles on the dotted line for the new TL curve and by the curve noted "Change in TL due to Damping." The coincidence between the change in TL and the change in damping above 4 kc is apparent from the figure and confirms the behavior we would expect in the region where the noise transmission is being governed by the free wave behavior.

In Fig. 5, we show TL behavior for a steel panel. In this case, damping was added at the edges by embedding one edge of the plate in sand. In this case, the plate is 1/16" thick with cross dimensions of 30" x 36". Again, we see at low frequencies a mass law behavior of the transmission curve breaking away near the critical frequency with damping becoming effective in increasing the TL in the free wave region of behavior.

IV. Vibration and Radiation of Bounded Panels

When the frequency is sufficiently low or the damping is sufficiently small so that the product $k\eta d$ falls below unity, then there is a reverberant build-up in the panel due to point source excitation. Although it is not so apparent, this is also true for acoustic excitation, i.e. the ratio of free wave to forced wave response for acoustic sources is inversely proportional to the product $k\eta d$, as shown in Eq. (6).

$$\frac{\text{free wave response}}{\text{forced wave response}} \sim \frac{1}{k_p \eta d} \qquad (6)$$

when $\lambda_f, \lambda_a < d$

under the general assumption that one is below the critical frequency of the panel and that the flexural and acoustic wavelengths are smaller than the mean free path. In this regime, the plate acts as a system with resonant modes of vibration. These modes have the amplitude versus frequency characteristic shown for the resonator in Fig. 1. The wavenumber response, on the other hand, for resonant panel modes is essentially like that shown in Fig. 6, where we have indicated the case of a mode resonating below the critical frequency. In this case, the wavenumber acceptance spectrum is not determined by the loss factor, but by the geometry of the panel. The amplitude of this acceptance spectrum, on the other hand, is dependent upon the damping and the frequency of excitation. Thus, in this case, both point source and acoustic excitation are damping limited because the whole response pattern of the mode rises and falls uniformly as a function of frequency. The acoustic wavenumbers tend to couple into the small wavenumber part of the modal spectrum whereas the point source excitation, as before, excites a broader range of response wavenumbers.

In this case, the response amplitudes and radiation behavior will be controlled by the total damping of the system and again the internal damping will have to be greater than the radiation damping for applied damping to result in reduced response or radiation. We again return to the criterion for internal damping given in Eq. (5). For resonant modes below the critical frequency, however, the radiation efficiency tends to be of the order of 10^{-1} to 10^{-2}. Thus, our criteria on the amount of required damping for aluminum plates in air and steel plates in water are reduced by a corresponding factor. This means that the internal damping for the aluminum plate must at least be of the order of 10^{-4} to 10^{-5} and the internal damping of the steel plate must exceed the range 10^{-2} to 10^{-3}. These values of damping are not difficult to achieve and we can expect, therefore, to be able to reduce response and radiation of sound in this regime by applied damping for either case.

For panel modes above the critical frequency, however, the radiation efficiency is again unity and the previous conclusions about the effectiveness of damping for the unbounded plate above the critical frequency will apply in this case as well.

V. Effect of Damping in Connected Structures

Damping is frequently applied to reduce the vibration of structures which are in turn tied into other structures, such as concrete walls which are attached into supporting walls, or aircraft panels which are tied into a larger structure. It will be necessary that any applied damping exceed the damping due to coupling if beneficial results are to be achieved. This is entirely analogous to the criterion that the internal applied damping must exceed the radiation damping in the case of panels "connected" to an acoustic field.

As an example of the relative roles of coupling and internal damping of the concrete wall which is attached into two other vertical walls and floor slabs, let us refer to Fig. 7. Here we show the reverberation time of a concrete wall in a building structure. At low frequencies, the losses can be explained by assigning an average absorption coefficient of 17% to the boundaries between the wall and its attachment. At higher frequencies, the observed losses can be accounted for by assuming a constant loss factor which is presumably associated with the wall material. At the lower frequencies, a substantial increase in internal damping would be necessary to observe any change in the total damping due to the very high damping already present due to the attachment of the structure to its environment.

VI. Conclusion and Acknowledgments

In this short discussion, it has only been possible to give a bird's-eye view of the expected role of damping in controlling vibration noise in very limited circumstances. It may be expected, however, that the general approach and conclusions can be extrapolated to many systems not explicitly dealt with here. The author would like to thank several of his colleagues at BBN; most particularly, Edward M. Kerwin, Jr., Ira Dyer and Gideon Maidanik for many helpful discussions of the problems presented here.

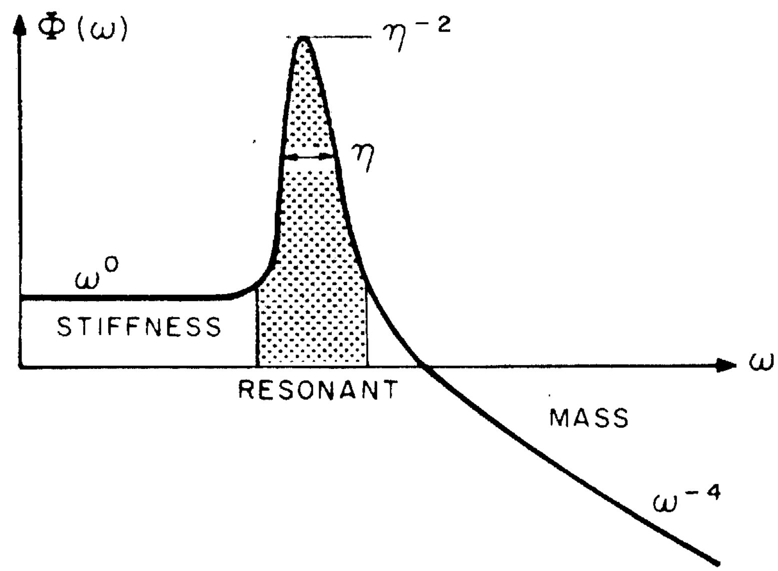


FIG. I RESPONSE CURVE OF SIMPLE RESONATOR

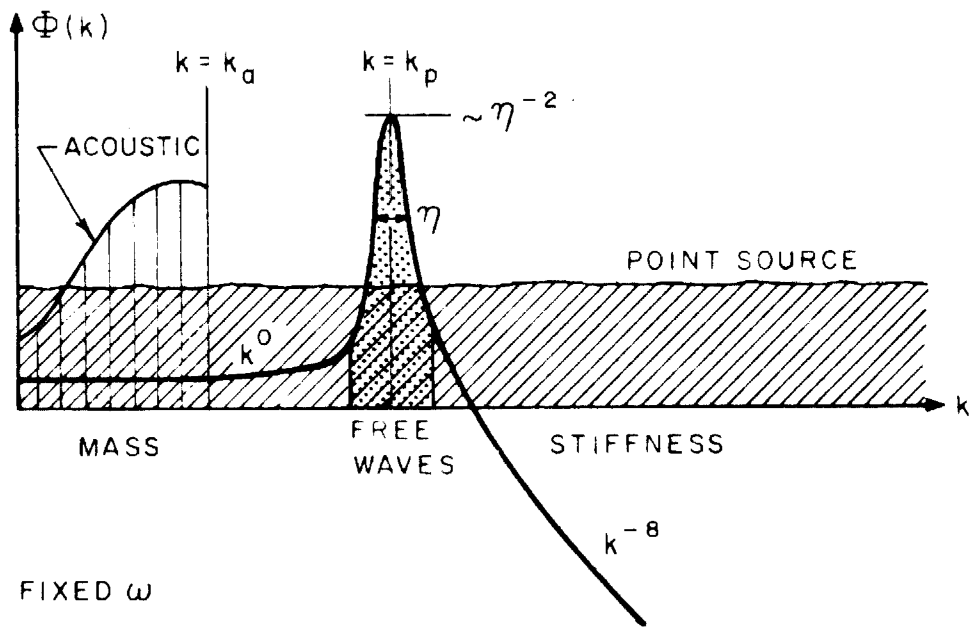


FIG. 2 WAVE NUMBER RESPONSE AND EXCITATION SPECTRA FOR UNBOUNDED PLATE BELOW THE CRITICAL FREQUENCY

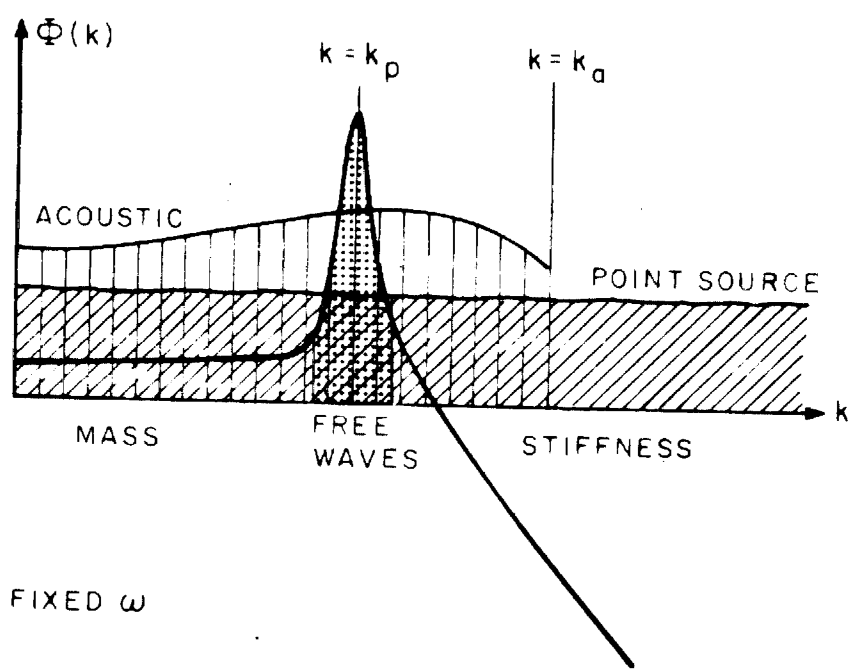


FIG. 3 RESPONSE AND EXCITATION SPECTRA FOR UNBOUNDED PLATE ABOVE CRITICAL FREQUENCY

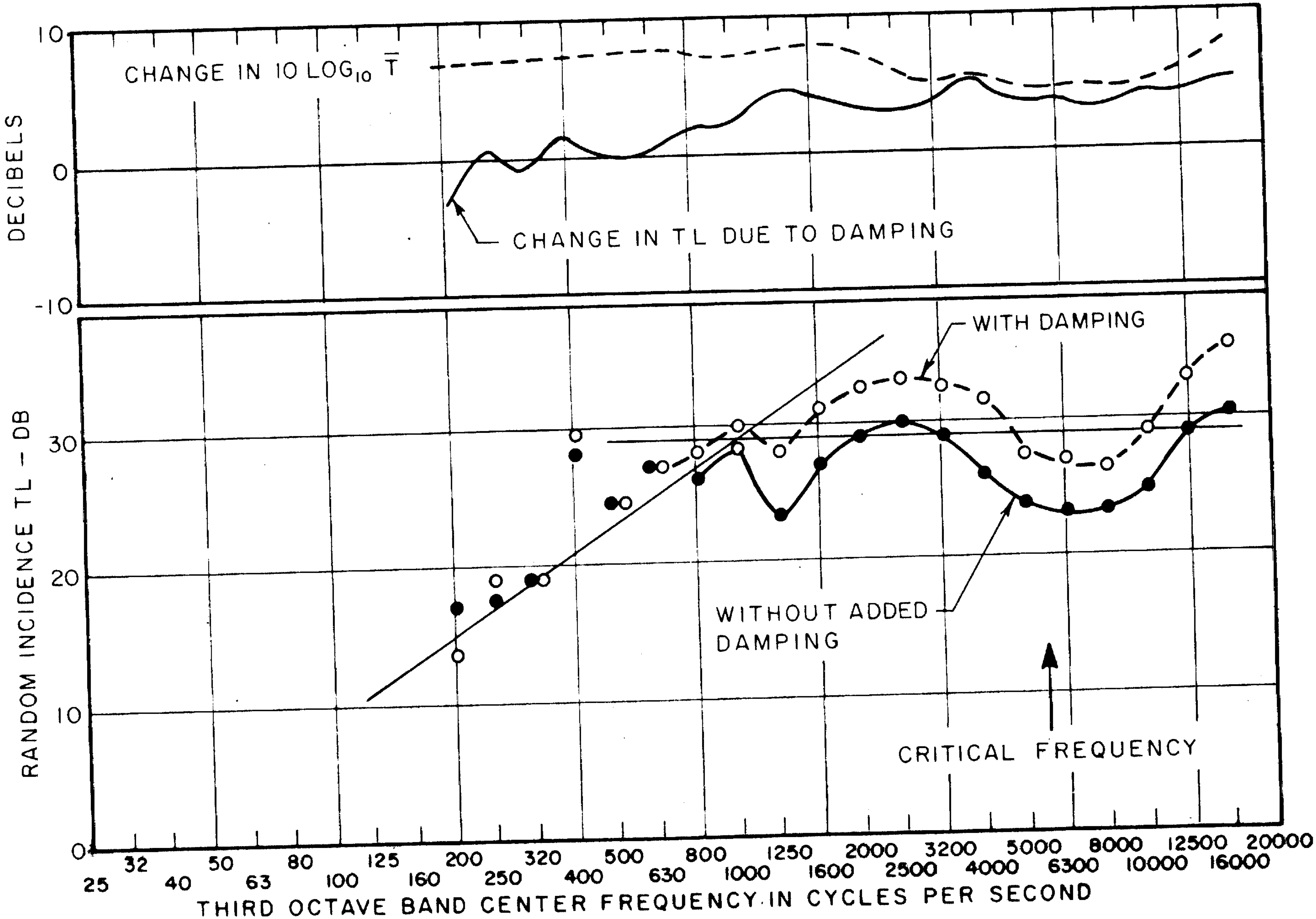


FIG. 4 MEASURED TL OF SCALE–MODEL WALL – 0.090" ALUMINUM PLATE, 12" x 34" CLAMPED AT EDGES

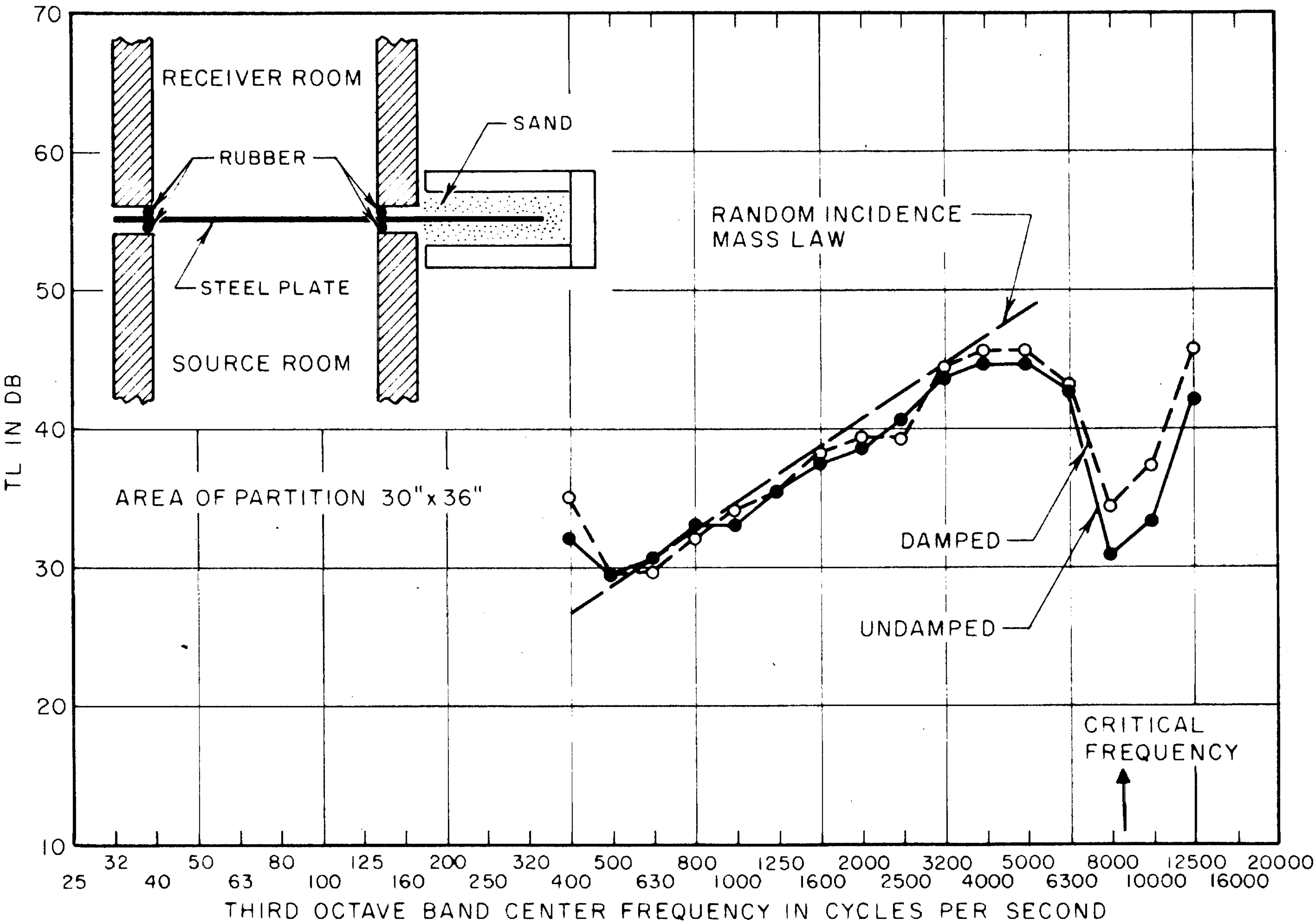
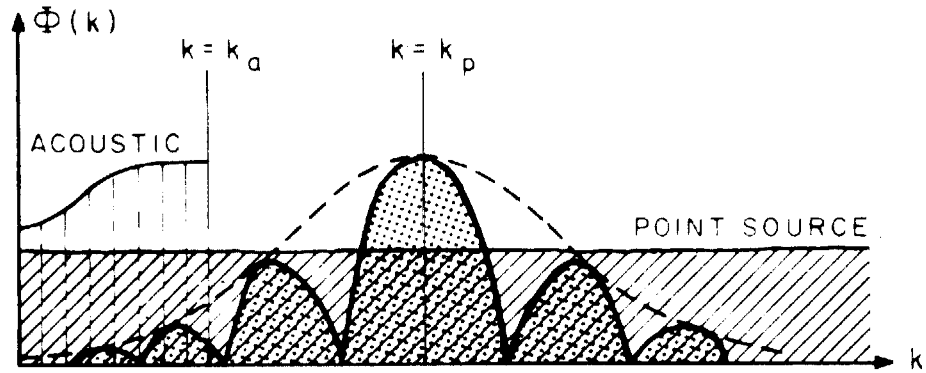


FIG. 5 TRANSMISSION LOSS OF A $\frac{1}{16}$" STEEL PLATE



FIXED ω

FIG. 6 RESPONSE AND EXCITATION SPECTRA FOR BOUNDED PLATE

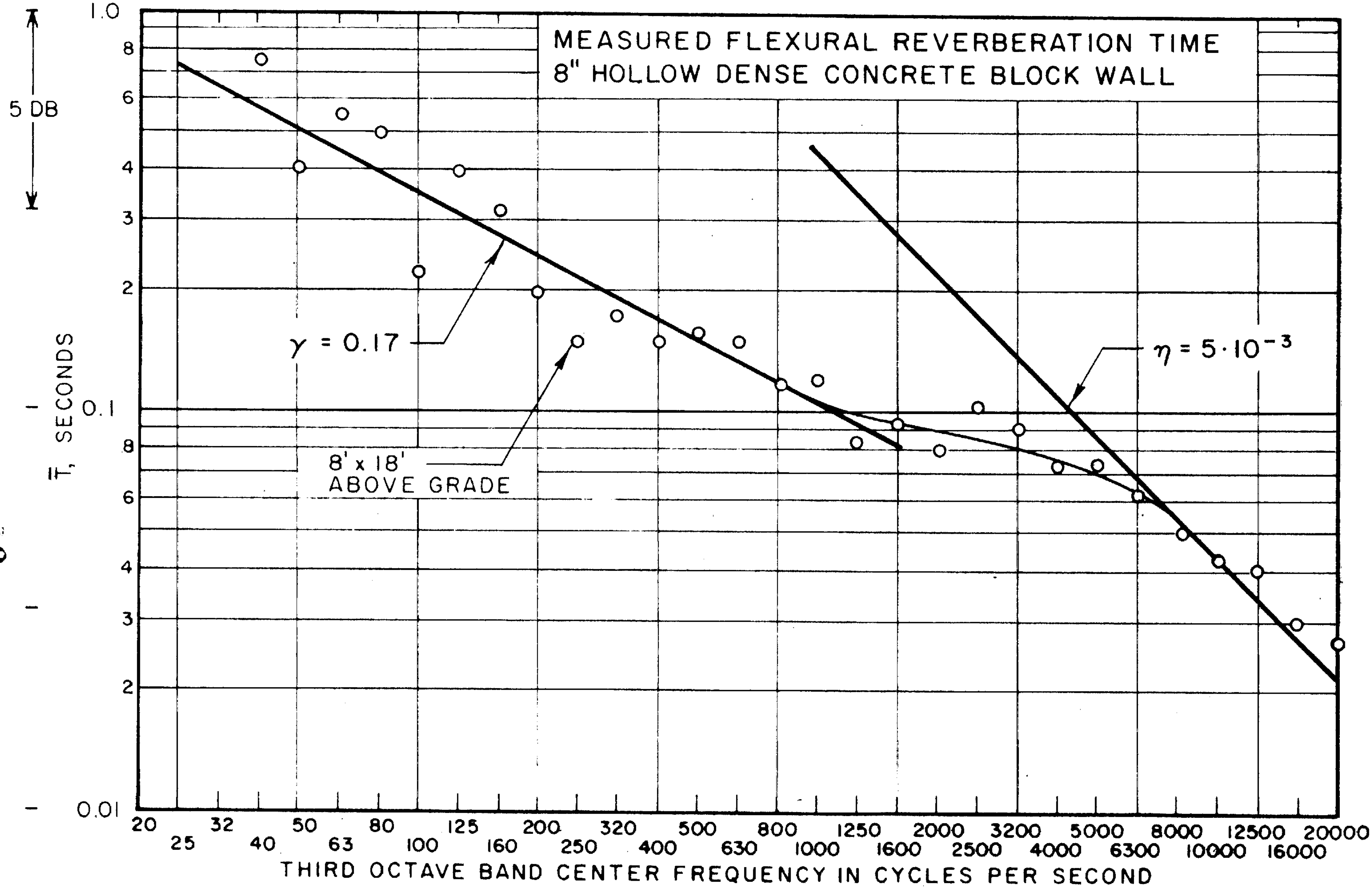


FIG. 7 BOUNDARY AND INTERNAL DAMPING REGIMES OF CONCRETE WALL